Longitudinal Mismatch in SCL as a source of Beam Halo

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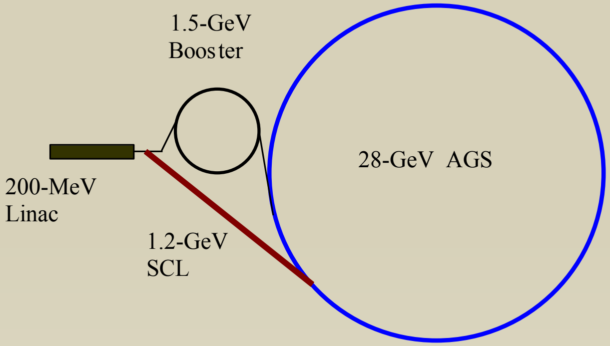
Abstract

- An advantage of a proton Super-Conducting Linac (SCL) is that RF cavities can be operated independently, allowing easier beam transport and acceleration. But cavities are to be separated by drifts long enough to avoid they couple to each other. Moreover, cavities are placed in cryostats that include inactive insertions for cold-warm transitions; and interspersed are warm insertions for magnets and other devices. The SCL is then an alternating sequence of accelerating elements and drifts. No periodicity is present, and the longitudinal motion is not adiabatic. This has the consequence that the beam bunch ellipse will tumble, dilute and create a halo in the momentum plane because of inherent nonlinearities. When this is coupled to longitudinal space-charge forces, it may cause beam loss with latent activation of the accelerator components.

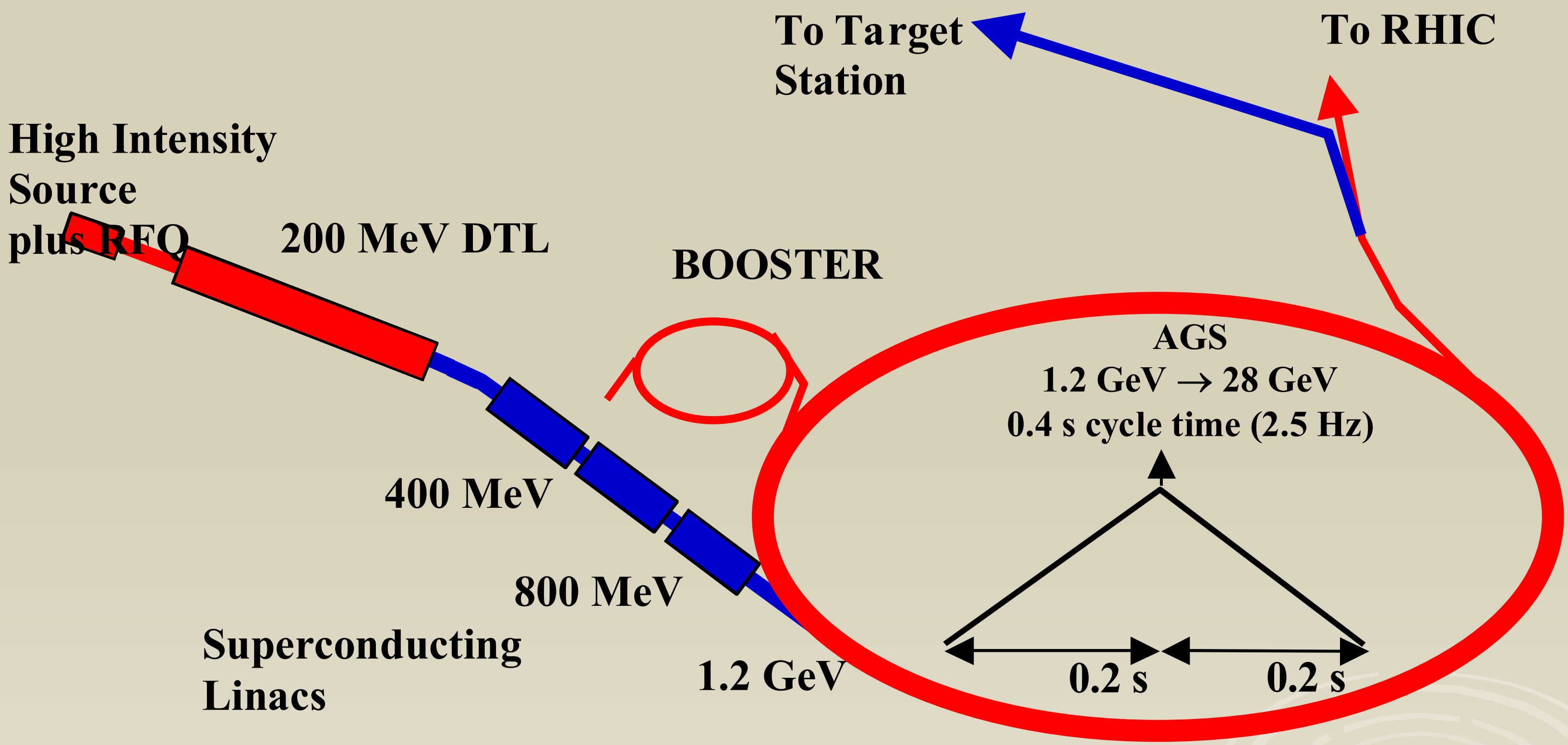
Super-Conducting Linacs

- At BNL we have several projects that require Super-Conducting Linacs (SCL)
- One of them is the AGS Upgrade for a proton average power of 1 MWatt

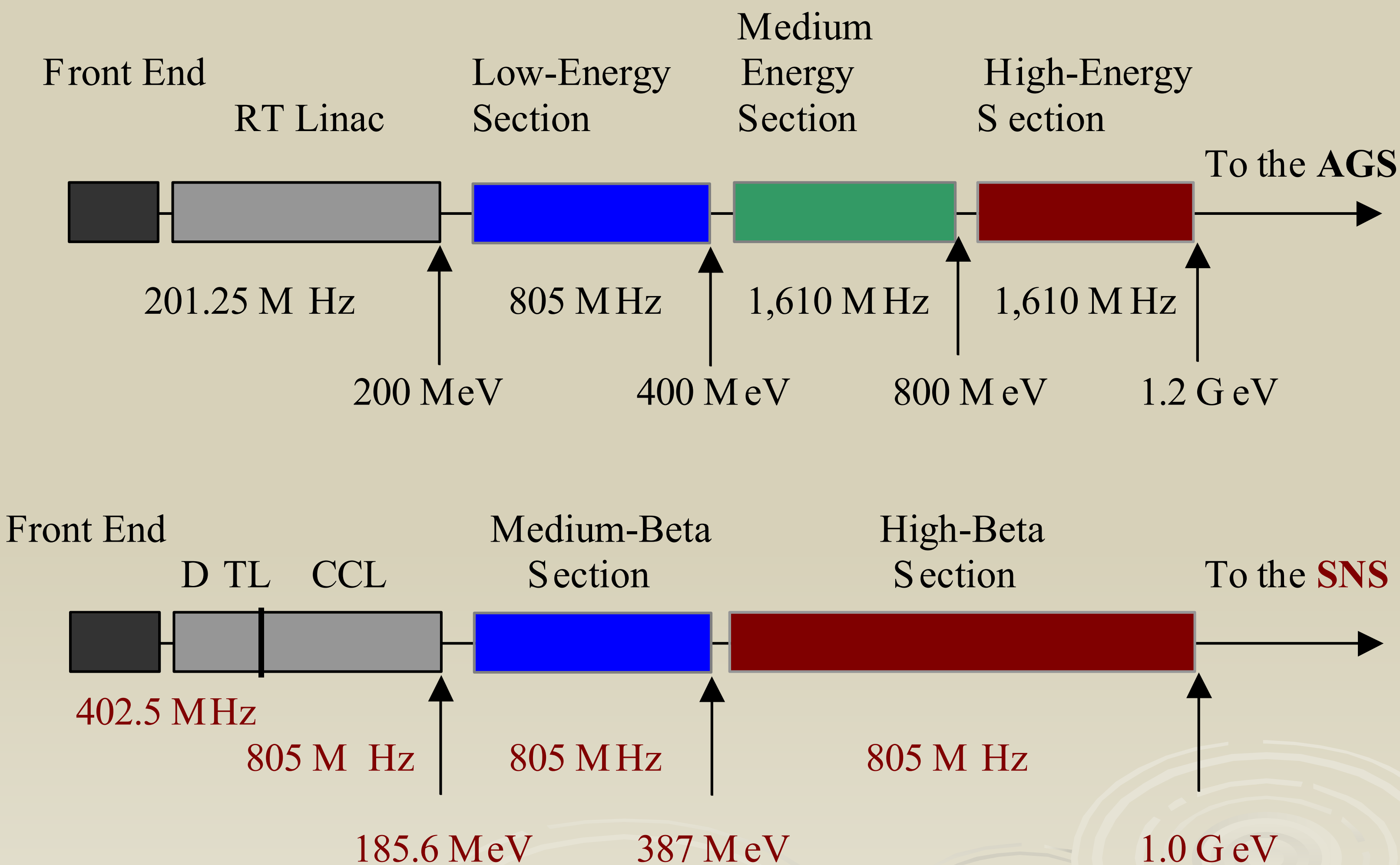
	AGS present	AGS upgrade	SNS
Kin. Energy, GeV	28	28	1.0
Protons 10^{14} / Cycle	0.67	0.89	1.04
Rep. Rate, Hz	1/3	2.5	60
Ave. Power, MW	0.10	1.0	1.0



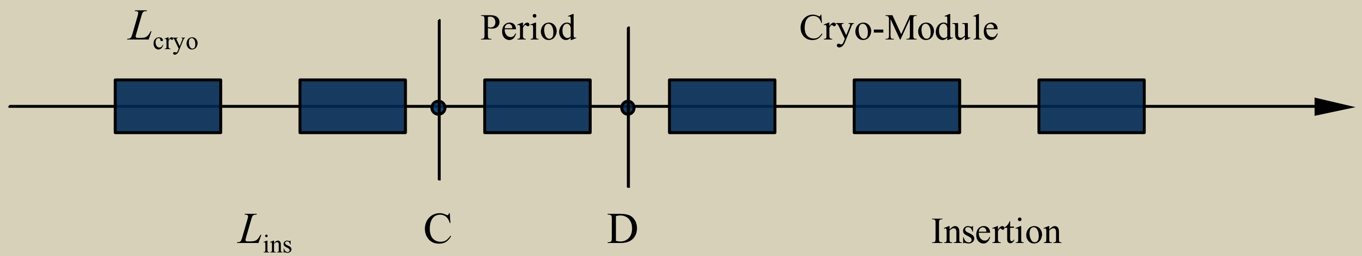
The 1.2-GeV SCL for the 1-MW AGS Upgrade

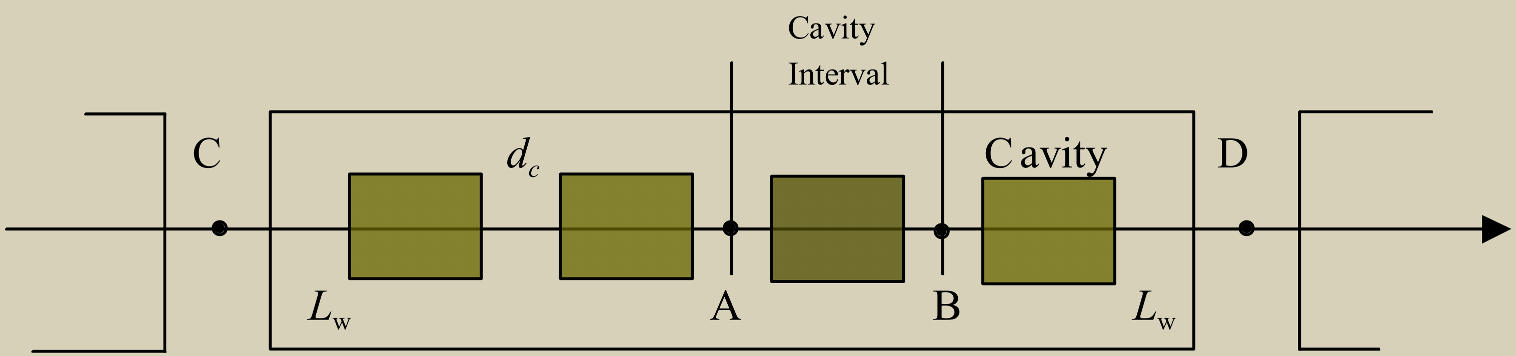


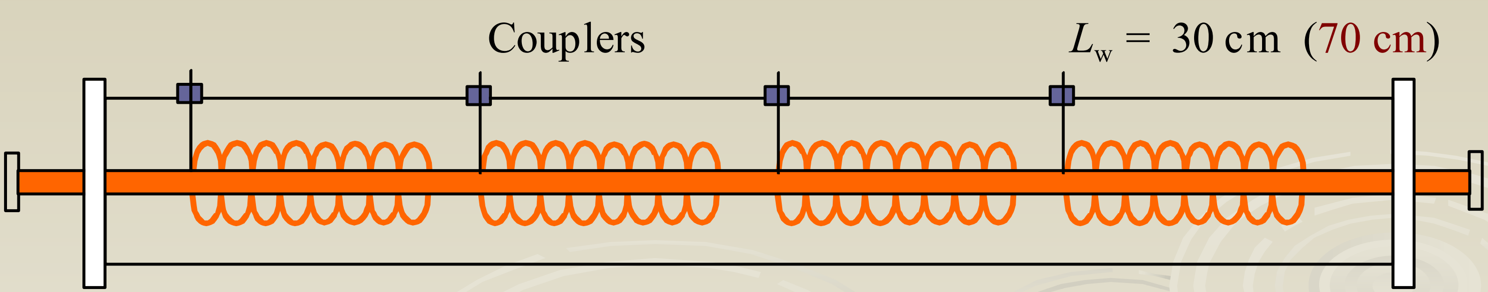
SCL Layout (AGS-Upgrade and SNS)



Any Energy Section of the SCL

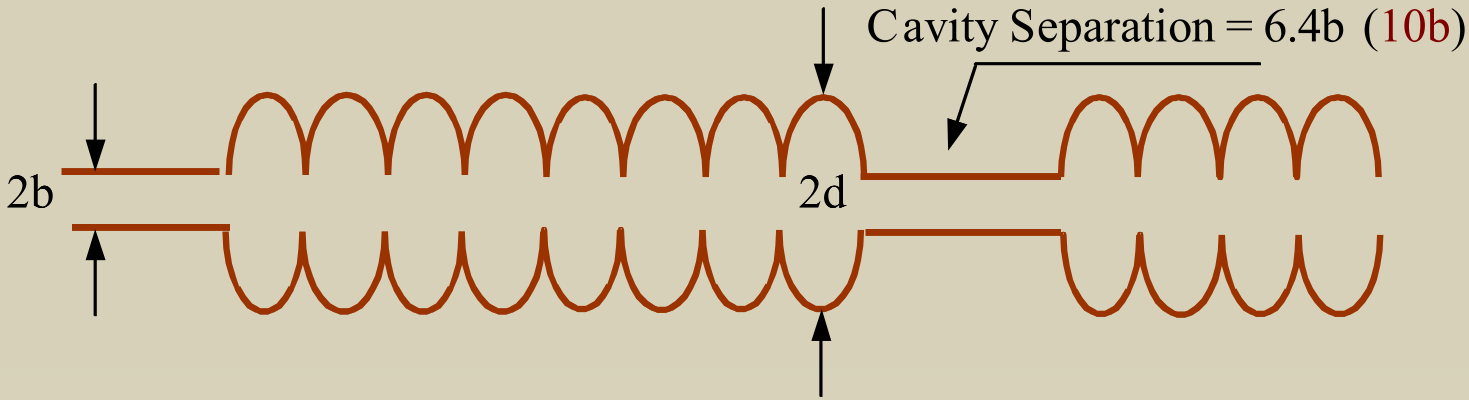






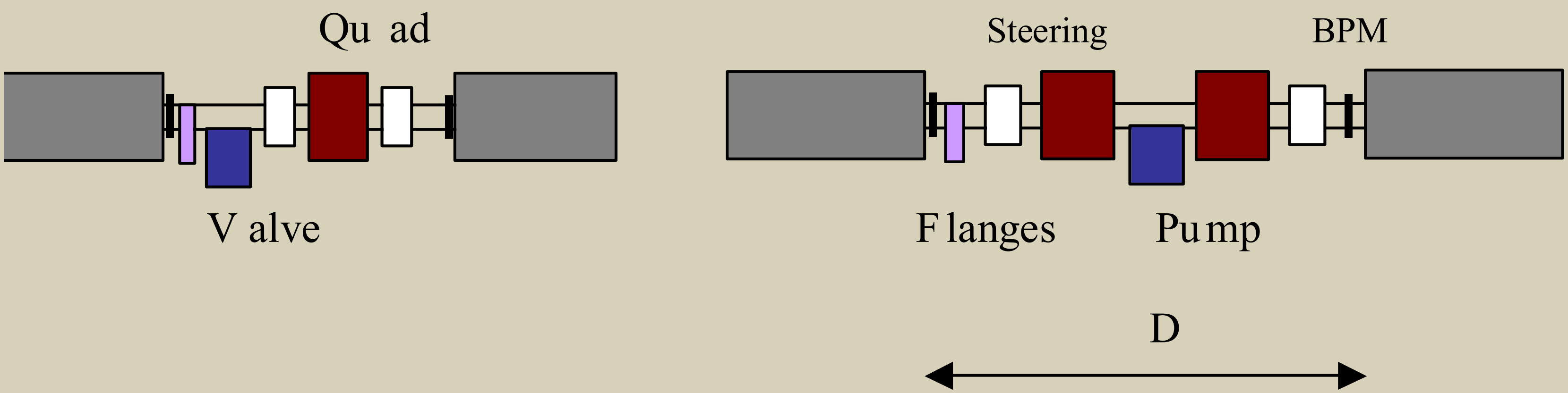
Cavity-Cell Arrangement

8 Cells per Cavity (6 in SNS)
4 Cavities per (Cryo-)Module (3 in Medium-β and 4 in High-β SNS)



2b	=	10 (8) cm	2d	=	34 cm	@	805 MHz
	=	5 cm		=	17 cm	@	1,610 MHz

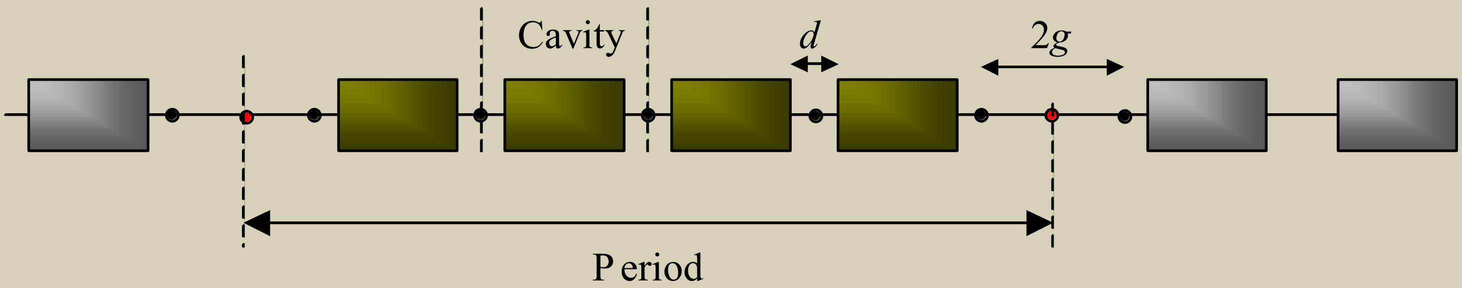
Warm Insertions



LE-Section	D	=	1.079 m	Singlet	Period	=	6.304 m
ME-Section			1.379 m (1.6 m)	Doublet (SNS)			4.708 (5.839)
HE-Section			1.379 m (1.6 m)	Doublet (SNS)			4.994 (7.891)

Quadrupole Length (effective) 30 cm (39 cm)

Thus a section of SCL is made of a alternating sequence of Drifts of different length (*g* and *d*) and Accelerating structures, namely Cavities with a number *N* of RF Cells.



AGS-Upgrade SCL

Section	LE	ME	HE
Injection Energy, MeV	200	400	800
Final Energy, MeV	400	800	1,200
RF Frequency, MHz	805	1,610	1,610
Average Axial Field, MV/m	13.4	29.1	29.0
No. of Periods	6	9	8
No. of Cavities/Period, *M*	4	4	4
No. of Cells/Cavity, *N*	8	8	8
Cell Length, cm	11.45	7.03	7.92
Cavity Length, cm	91.60	56.24	63.36
Cavity Separation, *d*, cm	32	16	16
Warm-Cold Transition, cm	30	30	30
Warm Insertion, cm	107.9	137.9	137.9
Period Drift, *g*, cm	67.95	90.95	90.95

Longitudinal Equations of Motion

- Time Delay τ □ t □ $t_{□}$
  Energy Difference ε □ □ □ $□_{□}$

□ $'$ $\equiv$ d / ds

- After linearization

$$\tau' = -\varepsilon / c\, \beta_s^3 \gamma_s^3 E_0 \qquad (1)$$

- Energy Equations

$$E' = eE_{acc} \cos(\omega t)$$

$$E_s' = eE_{acc} \cos(\omega t_s)$$

- After linearization

$$\varepsilon' = -\, eE_{acc} (\sin \phi_s)\, \omega \tau \qquad (2)$$

- For a segment of the accelerating structure, short enough to neglect the variation of $\beta_s^3 \gamma_s^3$, by combining (1) and (2)

$$\tau'' + K^2 \tau = 0$$

$$K = [eE_{acc} (-\sin \phi_s)\, \omega / c\, \beta_s^3 \gamma_s^3 E_0]^{1/2} = \Omega_s / c\beta_s$$

Matrix Method

- Drift of length ℓ

$$M_{drift}(\ell) = \begin{vmatrix} 1 & \ell \\ 0 & 1 \end{vmatrix} \qquad \begin{vmatrix} \tau \\ \tau' \end{vmatrix}$$

- Cavity of length L

$$M_{cavity} = \begin{vmatrix} \cos\theta & (\sin\theta)/K \\ -K\sin\theta & \cos\theta \end{vmatrix}$$

$$\theta = KL$$

- Define $\eta = d/L$

(1) Transfer Matrix for a Cavity

$$M_c = M_{drift}(d/2)\, M_{cavity}\, M_{drift}(d/2)$$

$$M_c = \begin{vmatrix} \cos\mu_c & \beta_c \sin\mu_c \\ -(\sin\mu_c)/\beta_c & \cos\mu_c \end{vmatrix} = \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix}$$

$$K\beta_c = (1 - \theta^2 \eta^2 / 4 + \theta\, \eta \cot\theta)^{1/2}$$

$$L \, tg\, \mu_c = \theta\, \beta_c / (\cot\theta - \theta\, \eta / 2)$$

(2) Transfer Matrix for a Period

$$M_p \quad = \quad M_{drift}(g)\, M_c^{\,M}\, M_{drift}(g)$$

$$M_p \quad = \quad \begin{vmatrix} \cos \mu_p & \beta_p \sin \mu_p \\ -(\sin \mu_p) / \beta_p & \cos \mu_p \end{vmatrix} \quad = \quad \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix}$$

$$\beta_p = \quad [\beta_c^{\,2} \; - \; g^2 \; + \; 2 \,\square\, \beta_c \cot_{.}\; M \,\mu_{c.}]^{\,1/2}$$

$$L\, tg\, \mu_c \quad = \quad \beta_p / [\beta_c \cot_{.}\; M \,\mu_{c.} \; - \; g]$$

- **Note**: the use of the transfer matrices M_p and M_c gives a good approximation when the energy gain across one Cavity section and, eventually, one Period is a small fraction of the energy in entrance.

- **Beam Bunch Dimensions**

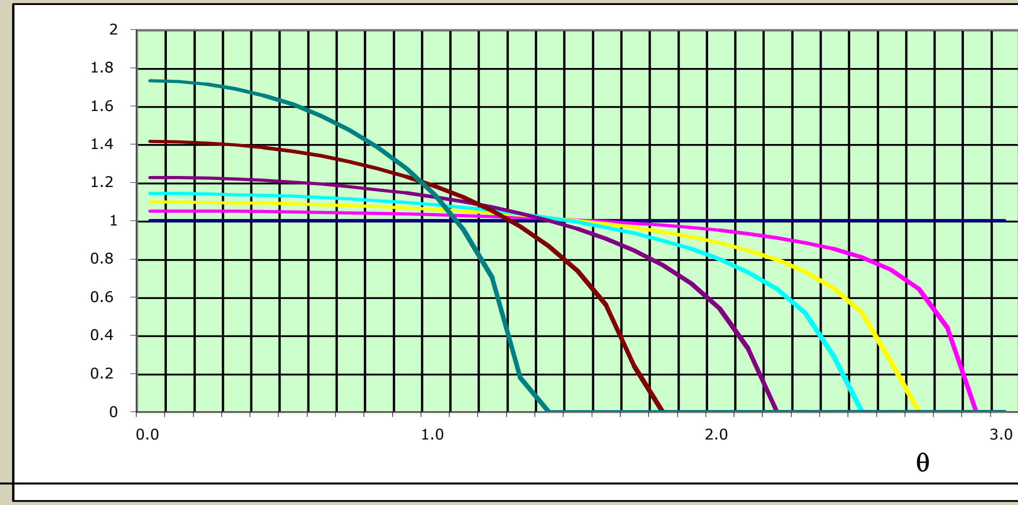
Bunch Area	S	=	$\pi \, \Delta\tau \, \Delta\varepsilon$	
Bunch (Half) Length	$\Delta\tau$	=	$(S_n \, \beta_{\square p} / \pi)^{1/2}$	(sec)
	S_n	=	$S / c \, \beta_s^3 \, \gamma_s^3 \, E_0$	(sec – sec/m)

- **Stability of Motion**

| Trace of M_c and M_p |	< 2
| $\cos \mu_{c,\,p}$ |	real and < 1
$\beta_{\square\, p}$	real and > 0

Stability Diagrams

- Plot of $K\beta_{\square}$ versus θ for various values of η



- Plot of $\beta_p / \beta_{\square}$ vs. $M\mu_c$ for several values of $g/\beta_{\square}$

Plot of max value of θ versus η

Mismatch of Motion

- We have assumed that *K* does not change across a *period*. This is justified if the acceleration rate is not too high, and the energy change is only a small fraction of the total kinetic energy.

- Exact matching is achieved by requiring that the amplitude value β_p remains constant from one period to the next. In turn that requires that also β_c and μ_c per cavity interval remain unchanged. That is the rotation angle θ and the restoring parameter *K* are also constant. For given frequency ω and RF phase ϕ_s, we derive the following condition for exact matching

  $$E_{acc} / \beta_s^3 \gamma_s^3 \quad = \quad \text{constant}$$

  A difficult condition to satisfy!

- A simpler mode of operation assumes a constant energy gain per *period*, so that the restoring parameter *K* will decrease with the beam energy. This will cause continuous mismatch of the particle motion, which is a beam bunch rotation (with consequent possible dilution) of an amount that depends on the acceleration rate, and on the length of the drift insertions (*d* and *g*).

- In analogy to the conventional approach used to describe the transverse motion, a beam bunch can be made to correspond to an ellipse in the phase space (τ, τ'). Between periods, the ellipse is described by the amplitude β_p and the inclination α_p.

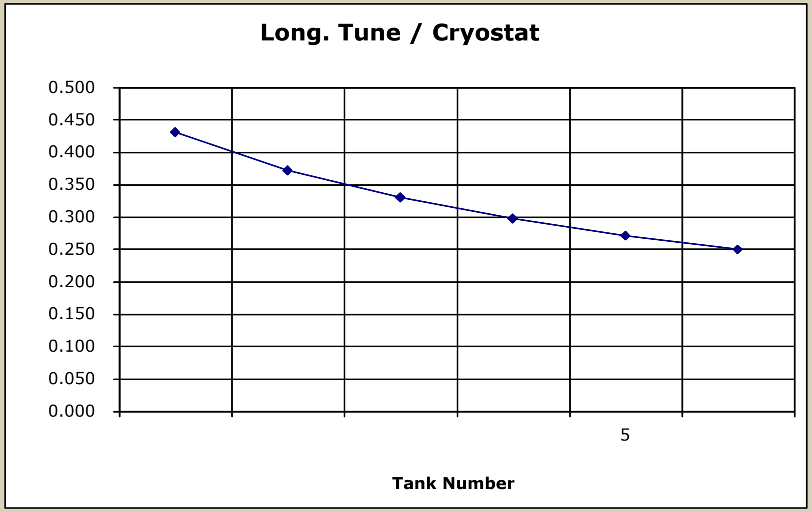
Mismatch of Motion

- To estimate the amount of mismatch as the motion progress, we assume that the beam bunch is exactly matched at the entrance of the SCL section, where $\alpha_p = 0$. It is well know then how to estimate the bunch ellipse rotation, dilation or contraction from one period to the next with the transformation

$$\begin{vmatrix} \beta_p \\ \alpha_p \\ \gamma_p \end{vmatrix}_2 = \begin{vmatrix} m_{11}^2 & -2\, m_{11}\, m_{12} & m_{12}^2 \\ -m_{21}\, m_{11} & 1 + 2\, m_{12}\, m_{21} & -m_{12}\, m_{22} \\ m_{21}^2 & -2\, m_{22}\, m_{21} & m_{22}^2 \end{vmatrix} \begin{vmatrix} \beta_p \\ \alpha_p \\ \gamma_p \end{vmatrix}_1$$

$$\beta_p\, \gamma_p = 1 + \alpha_p^2$$

Low-Energy Section of SCL for 1-MW AGS Upgrade

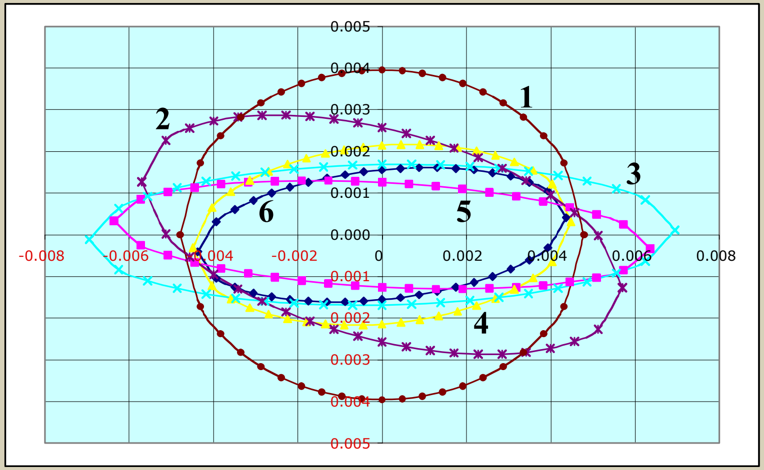


Conclusion

- Usually, our perception of a beam moving down an accelerator is made of bunches having the longitudinal shape of unchanging upright ellipses. This may be indeed a good approximation in circular accelerators where the energy gain per turn is small, but is not correct in the case of linear accelerators where the acceleration rate is considerably higher and the beam energy changes over short periods of length.

- Moreover, active accelerating cavities are separated by drifts of various lengths that introduce an intrinsic mismatch by which beam bunches actually rotate, elongate and contract over considerably short periods of length of only few meters. This has an analog with the transverse motion where also the betatron emittance ellipse changes continuously but periodically. The longitudinal motion nevertheless has no intrinsic periodicity and it is continuously mismatched from one location to the next. Our concern is that the continuous tumbling rapidly changing in a SCL of the bunches may lead to the creation of longitudinal halos accompanied by latent beam losses.

- Operation of high-power SCL in the GeV range needs to be demonstrated. There are several proton SCL projects being proposed, but only one is presently under construction (SNS) and will be soon, hopefully, in operation.



$$S_n/\pi = \gamma_{\square}\,\tau^2 + 2\alpha_{\square}\,\tau\,\tau' + \beta_{\square}\,\tau'^2$$

$$\tau' / \tau_0' = -\alpha\ \tau / \tau_0 + [\,1 - (\tau / \tau_0)^2\,]^{1/2}$$